

# Fermi-liquid regime of the mesoscopic Kondo problem

Denis Ullmo,<sup>1,2,\*</sup> Dong E. Liu,<sup>3</sup> Sébastien Burdin,<sup>4,5</sup> and Harold U. Baranger<sup>3</sup>

<sup>1</sup>Univ Paris-Sud, LPTMS, UMR8626, 91405 Orsay, France, EU

<sup>2</sup>CNRS, 91405 Orsay, France, EU

<sup>3</sup>Duke University, Box 90305, Durham, NC 27708-0305, USA

<sup>4</sup>Univ Bordeaux, LOMA, UMR 5798, F-33400 Talence, France, EU

<sup>5</sup>CNRS, LOMA, UMR 5798, F-33400 Talence, France, EU

(Dated: October 17, 2012)

We consider the low temperature regime of the mesoscopic Kondo problem, and in particular the relevance of a Fermi-liquid description of this regime. Using two complementary approaches – a mean field slave fermion approximation on the one hand and a Fermi-liquid description “à la Nozières” supplemented by an argument of separation of scale on the other hand – we show that they both lead to (essentially) the same quasi-particle spectra, providing in this way a strong indication that they both give the correct physics of this regime.

PACS numbers: 71.10.Ay, 75.20.Hr, 73.21.La

The concept of a Fermi liquid,<sup>1,2</sup> introduced by Landau to describe the low temperature properties of He<sup>3</sup> (above the superfluid transition), turned out to be one of the most effective of condensed matter physics. In its original phenomenological version, its formulation is based on the assumption that, in spite of possibly rather strong bare interactions, a set of interacting fermions may (under certain conditions) behave at low energy as weakly interacting quasi-particles. For Landau Fermi-liquids, such as the ones describing He<sup>3</sup> or the electron liquid for  $d > 1$ , a modern formulation in terms of renormalization group analysis<sup>3</sup> provides a rigorous basis for this hypothesis. The main strength of the Fermi-liquid approach, however, is that it makes it possible to take full advantage of the symmetries of the problem under consideration, and of the fact that the temperatures or energies considered are much smaller than all “natural” energy scales of the problem, allowing for perturbative expansions near the Fermi energy. In this way, the quasi-particles as well as their weak mutual interactions can be characterized by a small number of parameters, quite often fixed in practice by measuring a few relevant quantities. From those, the full behavior of the system can be determined.

Another kind of Fermi liquid, which is going to be our main concern in this paper, is the one introduced by Nozières<sup>1,4,5</sup> to describe the low energy physics of the Kondo problem.<sup>6</sup> In its simplest version, referred to as the  $s$ - $d$  (or simply Kondo) model, this consists in the (local) interaction of a gas of non-interacting fermions with a spin one half. The corresponding Hamiltonian then reads

$$H_K = \sum_{\alpha\sigma} \epsilon_\alpha \hat{c}_{\alpha\sigma}^\dagger \hat{c}_{\alpha\sigma} + H_{\text{int}} + g\mu_B B \cdot S_z, \quad (1)$$

where  $\hat{c}_{\alpha\sigma}^\dagger$  creates a particle with energy  $\epsilon_\alpha$ , spin  $\sigma$  and wave-function  $\varphi_\alpha(\mathbf{r})$ , and the interaction with the impurity is expressed as

$$H_{\text{int}} = J_0 \mathbf{S} \cdot \mathbf{s}(\mathbf{r}_0) \quad (2)$$

with  $J_0 > 0$  the coupling strength,  $\hbar \mathbf{S} = \hbar(S_x, S_y, S_z)$  a quantum spin 1/2 operator ( $S_i$  is half of the Pauli matrix  $\sigma_i$ ),  $\hbar \mathbf{s}(\mathbf{r}_0) = (\hbar/2) \hat{\Psi}_\sigma^\dagger(\mathbf{r}_0) \boldsymbol{\sigma} \hat{\Psi}_\sigma(\mathbf{r}_0)$  the spin density of

the electron gas at the impurity position  $\mathbf{r}_0$ , and  $\hat{\Psi}_\sigma^\dagger(\mathbf{r}_0) = \sum_\alpha \varphi_\alpha(\mathbf{r}_0) \hat{c}_\alpha^\dagger$ . Finally, with the last term on the right hand side of Eq. (1) we consider also the possibility that the impurity spin is coupled to a magnetic field  $\mathbf{B} = B\hat{z}$ , with  $g$  and  $\mu_B$  the corresponding Landé factor and Bohr magneton.

In the original formulation of the Kondo problem, the magnetic impurity was assumed to be an actual impurity (e.g. Fe) in a bulk piece of metal (e.g. Au), in which case the wave-functions  $\varphi_\alpha$  are plane waves and the spacing  $\Delta$  between the energy levels  $\epsilon_\alpha$  can be assumed negligibly small and constant. The electron gas is then characterized by only two quantities: the local density of states  $\nu_0 = (\mathcal{A}\Delta)^{-1}$  ( $\mathcal{A}$  is the volume of the sample), and the bandwidth  $D_0$  of the spectrum. The Kondo problem assumes the dimensionless parameter  $\mathcal{J}_0 \equiv \nu_0 J_0$  to be small. In that case, it can be shown<sup>6</sup> that, beyond the Fermi energy which essentially defines the origin of the energies, the problem is characterized by a single energy scale, the Kondo temperature  $T_K$  [which in a one-loop perturbative approximation is given by  $T_K^{\text{1-loop}} = D_0 \exp(-1/(J_0 \nu_0))$ ]. The Kondo temperature  $T_K$  specifies the crossover between two different regimes: a weak interaction regime for  $T \gg T_K$  where, despite a renormalization of the coupling constant, the magnetic impurity is largely decoupled from the electron gas, and a strong interaction regime  $T \ll T_K$  where the magnetic impurity forms a singlet with the electrons of the gas.

Nozières,<sup>4</sup> using some physical arguments related to the large  $\mathcal{J}_0$  limit of the Hamiltonian (1) and evidence from numerical renormalization group calculations of Wilson,<sup>7</sup> proposed that the low temperature regime of the Kondo problem should be a Fermi liquid. As for Landau Fermi-liquids, symmetries, and the fact that the only energy scale is the Kondo temperature, makes it possible to essentially completely specify the properties of this Fermi liquid: the quasi-particles are characterized by a phase shift  $\delta_s(\epsilon_F) = s\pi/2$  at the Fermi energy ( $s = \pm 1$  is the sign of the spin), with a variation

$$\delta_s(\epsilon_F + \omega, B) = s\pi/2 + \omega/T_K - s(g\mu_B/2)B/T_K, \quad (3)$$

away from the Fermi energy and with a small magnetic field  $B$ . In addition to this phase shift, the magnetic impurity generates a weak effective interaction between the electrons of

the gas associated with virtual breaking of the Kondo singlet, and thus taking place only locally at the impurity,  $V_{\text{eff}} = (\pi\nu_0^2 T_K)^{-1} \hat{\Psi}_\uparrow^\dagger(\mathbf{r}_0) \hat{\Psi}_\uparrow(\mathbf{r}_0) \hat{\Psi}_\downarrow^\dagger(\mathbf{r}_0) \hat{\Psi}_\downarrow(\mathbf{r}_0)$ .

The progress made in the control and miniaturization of micro- or nano-structures has renewed interest in the Kondo problem, and in particular made relevant the question of what would happen if the magnetic impurity is connected to a *finite-size* fully-coherent electron bath rather than a bulk piece of material.<sup>8–23</sup> As a first consequence, the finite size of the bath implies a finite mean level spacing  $\Delta$ , and it becomes meaningful to discuss the individual properties of levels and wave-functions. Lack of translational invariance will furthermore be associated with interference effects, and thus mesoscopic fluctuations of the energy levels and wave-functions. These mesoscopic fluctuations will affect the Kondo physics, and in particular imply fluctuations of the Kondo temperatures.<sup>13,14,23–27</sup>

The question we address in this paper is how the low temperature Fermi-liquid description of the Kondo problem is modified in the presence of mesoscopic fluctuations. To simplify the discussion, we consider the (rather typical) situation where mesoscopic fluctuations exist for energies between the mean level spacing  $\Delta$  and some characteristic energy scale  $E_\text{fl}$  (such as the Thouless energy), and furthermore assume this latter energy scale to be smaller than the Kondo temperature. Thus as one lowers the temperature, the Kondo singlet is formed before any mesoscopic fluctuations set in, so that one does not expect any fluctuations of the Kondo temperature itself (the interrelation between the fluctuations of the Kondo temperature and the mesoscopic fluctuations of the quasi-particles is left for future studies).

As always when one tries to develop a Fermi-liquid description for a mesoscopic system, a conceptual difficulty arises. As mentioned above, what makes a Fermi liquid description effective is not so much the derivation of its parameters from a microscopic calculation, but rather the fact that because there are only a small number of energy scales at play (and usually a high degree of symmetry), the number of parameters required to fully describe the system is rather small. For mesoscopic systems, however, one has fluctuations at all scales between  $\Delta$  and  $E_\text{fl}$ . It is therefore a priori hopeless to obtain a parameterization with only a small number of parameters.

In such circumstances, one possible approach to the Fermi-liquid regime of the mesoscopic Kondo problem is to use a mean field slave fermion approach. Up to some well understood limitations, such a mean field approximation is known to provide a good description of the Fermi-liquid regime of the bulk Kondo problem,<sup>6</sup> and it is reasonable to expect that this will remain the case for its mesoscopic version.

Let us summarize briefly the principle of the mean field approximation scheme.<sup>28</sup> Starting from the Hamiltonian  $H_K$  Eqs. (1)-(2), one introduces a representation of the spin  $\mathbf{S}$  in terms of Abrikosov fermions  $\hat{f}_\sigma^\dagger$  with spin  $\sigma = \uparrow, \downarrow$  and writes the Kondo part of the Hamiltonian as

$$H_{\text{int}} = \frac{J_0}{2} \sum_{\sigma\sigma'} f_\sigma^\dagger f_{\sigma'} \hat{\Psi}_{\sigma'}^\dagger(\mathbf{r}_0) \hat{\Psi}_\sigma(\mathbf{r}_0) - \frac{J_0}{4} \sum_\sigma \hat{\Psi}_\sigma^\dagger(\mathbf{r}_0) \hat{\Psi}_\sigma(\mathbf{r}_0). \quad (4)$$

One furthermore needs to impose the constraint

$$\hat{f}_\uparrow^\dagger \hat{f}_\uparrow + \hat{f}_\downarrow^\dagger \hat{f}_\downarrow = 1, \quad (5)$$

which is done through a Lagrange multiplier  $\epsilon_0$  in the action and amounts in practice to the substitution

$$H_{\text{imp}} \mapsto H_{\text{imp}} + \epsilon_0 \sum_\sigma \hat{f}_\sigma^\dagger \hat{f}_\sigma.$$

This fermionic representation is exact. The mean field approximation consists in replacing the quartic part of the Kondo term by an effective quadratic term  $\sum_{\sigma\sigma'} \hat{f}_\sigma^\dagger \hat{f}_{\sigma'} \hat{\Psi}_{\sigma'}^\dagger(\mathbf{r}_0) \hat{\Psi}_\sigma(\mathbf{r}_0) \mapsto \sum_{\sigma\sigma'} [\hat{f}_\sigma^\dagger \hat{\Psi}_\sigma(\mathbf{r}_0) \langle \hat{\Psi}_{\sigma'}^\dagger(\mathbf{r}_0) \hat{f}_{\sigma'} \rangle + \text{h.c.}]$  where the mean value  $\langle \dots \rangle$  is computed self consistently. The mean field Hamiltonian obtained in this way is a resonant level model:

$$H_{\text{MF}} = \sum_\sigma \left[ \left( \sum_\alpha \epsilon_\alpha \hat{c}_{\alpha\sigma}^\dagger \hat{c}_{\alpha\sigma} \right) + \epsilon_{0\sigma} \hat{f}_\sigma^\dagger \hat{f}_\sigma + \left( v^* \hat{f}_\sigma^\dagger \hat{\Psi}_\sigma(\mathbf{r}_0) + v \hat{\Psi}_\sigma^\dagger(\mathbf{r}_0) \hat{f}_\sigma \right) \right] \quad (6)$$

where we define  $\epsilon_{0\sigma} \equiv \epsilon_0 + sg\mu_B B/2$ . To fix the parameters of the resonant level model, there are two self-consistency conditions,

$$v = \frac{J_0}{2} \sum_\sigma \langle \hat{f}_\sigma^\dagger \hat{\Psi}_\sigma(\mathbf{r}_0) \rangle, \quad (7)$$

$$1 = \sum_\sigma \langle \hat{f}_\sigma^\dagger \hat{f}_\sigma \rangle; \quad (8)$$

note that the second one consists in treating the constraint Eq. (5) only on average. This mean field treatment has been used to study the mesoscopic Kondo problem in Refs. 21–23.

Under our assumption that the Kondo temperature is larger than the scale  $E_\text{fl}$  below which mesoscopic fluctuations occur, we expect that the parameters  $v$  and  $\epsilon_0$  are the same as for the bulk analogue of our system. A detailed analysis of the fluctuations of the mean field parameters shows indeed that their relative variance goes to zero as  $T_K$  becomes much larger than the mean level spacing  $\Delta$ .<sup>13,23,27</sup> Therefore, in the low temperature regime,  $\epsilon_0$  is fixed to the Fermi energy  $\mu$  (that we take equal to zero), and the low temperature limit of the resonance width  $\Gamma(T) \equiv \pi\nu_0 |v|^2$  can be interpreted as the Kondo temperature. More precisely,  $\Gamma(0) = a_k T_K^{\text{MF}} = \frac{1}{2} T_K^{1\text{-loop}}$  ( $a_k \simeq 1.133 \dots$ ), where  $T_K^{\text{MF}}$  and  $T_K^{1\text{-loop}}$  are respectively the mean field and one-loop approximations to the Kondo temperature. Now as can be easily shown (see e.g. Ref. 23) the eigenenergies  $\lambda_{\beta\sigma}$  of the resonant level are the solutions of the equation

$$\sum_\alpha \frac{|\varphi_\alpha(\mathbf{r}_0)|^2}{\lambda - \epsilon_\alpha} = \frac{\lambda - \epsilon_{0\sigma}}{|v|^2} = \frac{\pi\nu_0}{\Gamma} (\lambda - \epsilon_{0\sigma}). \quad (9)$$

In Refs. 22 and 23, this equation was the starting point for analyzing the spectral fluctuations of the Landau quasi-particle energies of the low temperature mesoscopic Kondo problem.

What we intend to show here is that Eq. (9), or at least its close analog, can be derived directly from a Fermi-liquid analysis “à la Nozières” of the mesoscopic Kondo problem. The approach we follow is similar in spirit to the way a Landau Fermi-liquid description is obtained for a mesoscopic electron liquid, relevant in the context of ballistic or diffusive quantum dots.<sup>29</sup> Indeed in that case, too, the existence of fluctuations at all energy scales between the Thouless energy and the mean level spacing seems to prevent the description of the problem in terms of a few parameters. This problem has been “solved” using essentially an argument of separation of scales: if the screening length of the Coulomb interaction is much smaller than the size of the dot, the renormalization of the mass and of the interactions (short length scale) are the same as in the bulk, while the confinement effects (large length scale) are obtained from a self-consistent potential derived from a Thomas-Fermi treatment. Note however that this common wisdom way of dealing with a mesoscopic Landau Fermi liquid does not rely on an analytic derivation (see for instance the renormalization group approach proposed in Ref. 29 and the difficulty with that approach discussed in Ref. 30), but it has been effective in interpreting most experimental data.

This is the philosophy we would like to follow for the Fermi-liquid description of the mesoscopic Kondo problem. To remain general, we assume only that, as above, our mesoscopic system is characterized by the energy scale  $E_{\text{fl}}$ , below which mesoscopic fluctuations take place but above which our electron bath behaves essentially as a bulk system. Thus, smoothing the density of states  $\rho_B(\epsilon) \equiv \sum_{\alpha} \delta(\epsilon - \epsilon_{\alpha})$  on the scale  $E_{\text{fl}}$  gives a “bulk”-like density of states  $\rho_0(\epsilon) \equiv \langle \rho_B \rangle_{E_{\text{fl}}}$  which shows no mesoscopic fluctuations and has only secular variations on classical scales (e.g. the Fermi energy). In the same way, we assume that a bulk-like wave-function probability  $\gamma_0(\epsilon) \equiv \langle |\varphi_{\alpha}|^2 \rangle_{E_{\text{fl}}}$  can be defined with variation only on classical scales. To help visualize the problem, one may think of the mesoscopic electron bath as a billiard, thus corresponding to the one particle Schrödinger Hamiltonian  $H_0 = -(\hbar^2/2m)\nabla^2$  inside some domain  $\mathcal{D}$  of area  $\mathcal{A}$  and typical size  $L$ . In that case,  $\rho_0(\epsilon)$  is the Weyl mean density of states,  $\gamma_0(\epsilon) = 1/\mathcal{A}$ , and  $E_{\text{fl}}$  is the Thouless energy  $E_{\text{Th}} = \hbar/\tau_{\text{fl}}$  with  $\tau_{\text{fl}} \equiv L/v_F$  the time of flight across the system ( $v_F$  is the Fermi velocity). We shall not, however, use any specific properties of billiards in what follows and shall furthermore assume that, unlike billiards, our system has a finite bandwidth  $D$  (which avoids unessential technical convergence problems and is in any case necessary for a properly defined Kondo problem).

Under the assumption that the Kondo temperature is larger than the Thouless energy, the impurity behaves at the local level as if it were placed in a bulk piece of material characterized by the density of states  $\rho_0$  and the wave-function probability  $\gamma_0(\epsilon)$ ; it is only at a longer length scale that the effects of the finiteness of the system are felt. To implement this intuitive idea, let us for a moment consider a localized static potential  $U(\mathbf{r}) = U_0\delta(\mathbf{r} - \mathbf{r}_0)$  (with  $\mathbf{r}_0$  the location of the impurity). [The divergences associated with a  $\delta$ -potential are avoided with a finite bandwidth.] Let us furthermore denote by  $G_B(\mathbf{r}, \mathbf{r}'; \omega)$  the Green function of our mesoscopic elec-

tron bath, and by  $G_0(\mathbf{r}, \mathbf{r}'; \omega)$  the corresponding “bulk-like” Green function. In particular we have

$$G_B(\mathbf{r}_0, \mathbf{r}_0; \omega) = \sum_{\alpha} \frac{|\varphi_{\alpha}(\mathbf{r}_0)|^2}{\omega - \epsilon_{\alpha} + i\eta} \quad (10)$$

$$G_0(\mathbf{r}_0, \mathbf{r}_0; \omega) = -i\pi\nu_0(\omega) + \Lambda(\omega), \quad (11)$$

with  $\nu_0(\omega) \equiv \gamma_0(\omega)\rho_0(\omega)$  the local density of states, and

$$\Lambda(\omega) = \gamma_0(\omega) \cdot \mathcal{P} \int d\epsilon \frac{\rho_0(\epsilon)}{\omega - \epsilon}. \quad (12)$$

The  $T$ -matrix of the static impurity in the bulk-like system is then given by

$$\begin{aligned} t(\omega) &= U + UG_0U + UG_0UG_0U + \dots \\ &= U \frac{1}{1 - G_0U}, \end{aligned} \quad (13)$$

while for the genuine mesoscopic system one has similarly

$$\begin{aligned} T(\omega) &= U + UG_BU + UG_BUG_BU + \dots \\ &= U \frac{1}{1 - G_BU}. \end{aligned} \quad (14)$$

Since  $U_0$ ,  $t(\omega)$ , and  $T(\omega)$  as well as  $G_0(\mathbf{r}_0, \mathbf{r}_0)$  and  $G_B(\mathbf{r}_0, \mathbf{r}_0)$  are just numbers, simple algebra leads to

$$T(\omega) = \frac{t(\omega)}{1 - \delta G(\omega)t(\omega)}, \quad (15)$$

with  $\delta G \equiv G_B - G_0$  the fluctuating part of the Green function. From this equation the full Green function  $G_B^{\text{tot}}$  including both the confinement and the impurity potential is given by

$$\begin{aligned} G_B^{\text{tot}}(\mathbf{r}, \mathbf{r}'; \omega) &= G_B(\mathbf{r}, \mathbf{r}'; \omega) \\ &+ G_B(\mathbf{r}, \mathbf{r}_0; \omega)T(\omega)G_B(\mathbf{r}_0, \mathbf{r}'; \omega). \end{aligned} \quad (16)$$

We argue that Eqs. (15) and (16) can be used to characterize the quasi-particles in the Fermi liquid description of the mesoscopic Kondo problem. Indeed, here the Green function  $G_B$  contains all the required information about the confinement properties (“long range”), and the Kondo physics (“short range”) is implemented by the  $T$ -matrix  $t(\omega)$ . At energies  $\omega$  sufficiently small compared to  $T_K$  so that inelastic processes are negligible,  $t(\omega)$  is related to the phase shift Eq. (3) through

$$t(\omega) = -\frac{1}{2i\pi\nu_0} [\exp(2i\delta_s(\omega)) - 1]. \quad (17)$$

The full Green function of the quasi-particles is therefore entirely determined through Eq. (16). In particular, the quasi-particles energies  $\lambda_{\beta}$  of the Fermi liquid are given by the poles of the  $T$ -matrix (15), and therefore fulfill the equation

$$G_B(\mathbf{r}_0, \mathbf{r}_0; \lambda) - G_0(\mathbf{r}_0, \mathbf{r}_0; \lambda) = 1/t(\lambda). \quad (18)$$

Noting that  $\text{Im}[1/t(\omega)] = \pi\nu_0$  and thus that away from the energies  $\epsilon_{\alpha}$  of the unperturbed problem the imaginary part

of Eq. (18) is automatically fulfilled, we see that the  $\lambda$ 's are therefore given as the solutions of

$$\sum_{\alpha} \frac{|\varphi_{\alpha}(\mathbf{r}_0)|^2}{\lambda - \epsilon_{\alpha}} - \Lambda(\lambda) = -\frac{\pi\nu_0 \sin(2\delta_s)}{1 - \cos(2\delta_s)} \simeq \frac{\pi\nu_0}{T_K} \left( \lambda - s \frac{g\mu_B}{2} B \right), \quad (19)$$

where in the last equality we have inserted the expression Eq. (3) for the phase shift and, to remain consistent, expanded to first order in  $1/T_K$ .

Comparing Eqs. (9) and (19), we see that they have the same structure, and basically contain the same qualitative content. Quantitatively they differ in two respects. First, the true Kondo temperature  $T_K$  in (19) is replaced in Eq. (9) by the resonance width  $\Gamma(0)$ , i.e., up to the factor  $a_k \simeq 1.133 \dots$  by the mean field approximation  $T_K^{\text{MF}}$  of the Kondo temperature. This is of course expected but should be kept in mind. Second, the term  $\Lambda(\lambda)$  in (19) is absent from (9). If the Fermi energy is in the middle of the band, this is of little importance as then  $\Lambda \sim 0$ . If this is not the case, however,  $\Lambda(\lambda)$  compensates the effect of states far from the Fermi energy, and its absence in (9) is certainly a limitation of the mean field approach. We see therefore that both the mean field and the Fermi liquid approaches have the same physical content as far as the spectrum of the quasi-particles is concerned, but one can expect better quantitative accuracy from the latter.

Finally, we can roughly estimate the range of energies for which the above neglect of inelastic processes is applicable. It is known that at low energy and temperature, the rate of inelas-

tic processes in the Kondo problem grows as the square of the deviation from the Fermi energy. This is the expected Fermi liquid result; it has been shown, for instance, that the imaginary part of the self-energy of the quasi-particles goes to zero as  $\omega^2$ ,  $\Im\{\Sigma(\omega)\} \propto \omega^2/T_K$ .<sup>31</sup> To extract the quasi-particle energies via the procedure outlined here, this inelastic rate must be much smaller than the level spacing,  $\omega^2/T_K \ll \Delta$ . Thus, the number of quasi-particle energy levels that are accurately treated by our argument,  $N \equiv \omega/\Delta$ , is roughly given by  $N \sim \sqrt{T_K/\Delta}$ . Since we are in any case assuming  $T_K \gg \Delta$ , this means that only levels near the center of the Kondo resonance can be captured.

To conclude, we have shown that the Fermi-liquid regime of the mesoscopic Kondo problem can be approached in two different ways: within a slave-fermion-mean-field framework, or more directly from a Fermi-liquid treatment “à la Nozières”. Limiting our discussion here to the quasi-particle spectra, we have seen that if the chemical potential is in the middle of the band, both approaches give the same result, except that the true Kondo temperature is replaced in the mean field approach by its mean field approximation (which is of course the best the mean field can provide). That both descriptions are essentially equivalent is a strong indication that they provide the correct physics of this regime. A definitive confirmation of this statement could be obtained for instance from a numerical renormalization group calculation for a few mesoscopic realizations.

The work at Duke was supported by US DOE, Office of Basic Energy Sciences, Division of Materials Sciences and Engineering under Grant No. de-sc0005237.

- 
- \* denis.ullmo@u-psud.fr
- <sup>1</sup> L. D. Landau, JETP **3**, 920 (1957).
  - <sup>2</sup> D. Pines and P. Nozières, *Theory of Quantum Liquids Vol. I*. (W. A. Benjamin, New York, 1966).
  - <sup>3</sup> R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
  - <sup>4</sup> P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
  - <sup>5</sup> P. Nozières and A. Blandin, J. Phys. France **41**, 193 (1980).
  - <sup>6</sup> A. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).
  - <sup>7</sup> K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).
  - <sup>8</sup> W. B. Thimm, J. Kroha, and J. von Delft, Phys. Rev. Lett. **82**, 2143 (1999).
  - <sup>9</sup> P. S. Cornaglia and C. A. Balseiro, Phys. Rev. B **66**, 115303 (2002); **66**, 174404 (2002); Phys. Rev. Lett. **90**, 216801 (2003).
  - <sup>10</sup> K. Kang and S.-C. Shin, Phys. Rev. Lett. **85**, 5619 (2000).
  - <sup>11</sup> I. Affleck and P. Simon, Phys. Rev. Lett. **86**, 2854 (2001).
  - <sup>12</sup> P. Simon and I. Affleck, Phys. Rev. Lett. **89**, 206602 (2002); Phys. Rev. B **68**, 115304 (2003).
  - <sup>13</sup> R. K. Kaul, D. Ullmo, S. Chandrasekharan, and H. U. Baranger, Europhys. Lett. **71**, 973 (2005).
  - <sup>14</sup> J. Yoo, S. Chandrasekharan, R. K. Kaul, D. Ullmo, and H. U. Baranger, Phys. Rev. B **71**, 201309(R) (2005).
  - <sup>15</sup> R. K. Kaul, G. Zaránd, S. Chandrasekharan, D. Ullmo, and H. U. Baranger, Phys. Rev. Lett. **96**, 176802 (2006).
  - <sup>16</sup> P. Simon, J. Salomez, and D. Feinberg, Phys. Rev. B **73**, 205325 (2006).
  - <sup>17</sup> R. G. Pereira, N. Laflorencie, I. Affleck, and B. I. Halperin, Phys. Rev. B **77**, 125327 (2008).
  - <sup>18</sup> S. Rotter, H. E. Türeci, Y. Alhassid, and A. D. Stone, Phys. Rev. Lett. **100**, 166601 (2008).
  - <sup>19</sup> S. Rotter and Y. Alhassid, Phys. Rev. B **80**, 184404 (2009).
  - <sup>20</sup> R. K. Kaul, D. Ullmo, G. Zaránd, S. Chandrasekharan, and H. U. Baranger, Phys. Rev. B **80**, 035318 (2009).
  - <sup>21</sup> R. Bedrich, S. Burdin, and M. Hentschel, Phys. Rev. B **81**, 174406 (2010).
  - <sup>22</sup> D. E. Liu, S. Burdin, H. U. Baranger, and D. Ullmo, Europhys. Lett. **97**, 17006 (2012).
  - <sup>23</sup> D. E. Liu, S. Burdin, H. U. Baranger, and D. Ullmo, Phys. Rev. B **85**, 155455 (2012).
  - <sup>24</sup> G. Zaránd and L. Udvardi, Phys. Rev. B **54**, 7606 (1996).
  - <sup>25</sup> S. Kettemann, in *Quantum Information and Decoherence in Nanosystems*, edited by D. C. Glatli, M. Sanquer, and J. T. T. Van (The Gioi Publishers, Hanoi, 2004) p. 259, (cond-mat/0409317).
  - <sup>26</sup> S. Kettemann and E. R. Mucciolo, Pis'ma v ZhETF **83**, 284 (2006), [JETP Letters **83**, 240 (2006); cond-mat:0509251].
  - <sup>27</sup> S. Kettemann and E. R. Mucciolo, Phys. Rev. B **75**, 184407 (2007).
  - <sup>28</sup> P. A. Lee, N. Nagaosa, and X. Wen, Rev. Mod. Phys. **78**, 17 (2006).
  - <sup>29</sup> I. L. Aleiner, P. W. Brouwer, and L. I. Glazman, Phys. Rep. **358**, 309 (2002).
  - <sup>30</sup> D. Ullmo, Rep. Prog. Phys. **71**, 026001 (2008).
  - <sup>31</sup> A. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993) pp. 113, 413–417.